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Effective Potential in Brane-World Scenarios

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We show that in brane-world scenarios with warped extra dimensions, the Casimir force due to bulk matter fields may be sufficient to stabilize the radion field ϕ. In particular, we calculate one loop effective potential for ϕ, induced by massless scalar fields propagating in the bulk in the Randall–Sundrum background. This potential has a local extremum, which can be a maximum or a minimum depending on the detailed bulk matter content. If the parameters of the background are chosen so that the hierarchy problem is solved geometrically, then the radion mass induced by Casimir corrections is hierarchically smaller than the *TeV*. Hence, in this important case, we must invoke an alternative mechanism (classical or nonperturbative) which gives the radion a sizable mass so as to make it compatible with observations.

1. INTRODUCTION

Recently, it has been suggested that theories with extra dimensions may provide a solution to the hierarchy problem (Antoniadis *et al.*, 1998; Arkani-Hamed *et al.*, 1998, 1999; Randall and Sundrum, 1999). The idea is to introduce a d-dimensional internal space of large physical volume $\mathcal{V}$, so that the the effective lower dimensional Planck mass $m_{\rm pl} \sim \mathcal{V}^{1/2} M^{(d+2)/2}$ is much larger than $M \sim TeV$—the true fundamental scale of the theory. In the original scenarios, only gravity was allowed to propagate in the higher dimensional bulk, whereas all other matter fields were confined to live on a lower dimensional brane. Randall and Sundrum (1999) (RS) introduced a particularly attractive model where the gravitational field created by the branes is taken into account. Their background solution consists of two parallel flat branes, one with positive tension and another with negative tension, embedded in a a five-dimensional anti-de Sitter (AdS) bulk. In this model, the hierarchy problem is solved if the distance between branes is about 37 times the AdS radius and we live on the negative tension brane. More recently, scenarios where additional fields propagate in the bulk have been considered (Altendorfer *et al.*, 2000; Gherghetta and Pomarol, 2000; Pomarol, 1999, 2000).

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In principle, the distance between branes is a massless degree of freedom, the radion field ϕ. However, in order to make the theory compatible with observations this radion must be stabilized (Charmousis *et al.*, 1999; Garriga and Tanaka, 1999; Goldberger and Wise, 1999, 2000; Tanaka and Montes, 2000). Clearly, all fields which propagate in the bulk will give Casimir-type contributions to the vacuum energy, and it seems natural to investigate whether these could provide the stabilizing force which is needed. Here, we shall calculate the radion one loop effective potential $V_{\text{eff}}(\phi)$ because of conformally coupled bulk scalar fields, although the result shares many features with other massless bulk fields, such as the graviton, which is addressed in (Garriga *et al.*, 2000). As we shall see, this effective potential has a rather nontrivial behavior, which generically develops a local extremum. Depending on the detailed matter content, the extremum could be a maximum or a minimum, where the radion could sit. For the purposes of illustration, here we shall concentrate on the background geometry discussed by Randall and Sundrum, although our methods are also applicable to other geometries, such as the one introduced by Ovrut *et al.* in the context of 11-dimensional supergravity with one large extra dimension (Lukas *et al.*, 1999). This report is based on a work done in collaboration with Jaume Garriga and Takahiro Tanaka (Garriga *et al.*, 2000).

Related calculations of the Casimir interaction amongst branes have been presented in an interesting paper by (Fabinger and Hořava, 2000). In the concluding section we shall comment on the differences between their results and ours.

2. THE RANDALL–SUNDRUM MODEL AND THE RADION FIELD

To be definite, we shall focus attention on the brane-world model introduced by (Randall and Sundrum, 1999). In this model the metric in the bulk is anti-de Sitter space (AdS), whose (Euclidean) line element is given by

$$ds^2 = a^2(z)\eta_{ab}\, dx^a\, dx^b = a^2(z)[dz^2 + d\mathbf{x}^2] = dy^2 + a^2(z)\, d\mathbf{x}^2. \tag{2.1}$$

Here $a(z) = \ell/z$, where ℓ is the AdS radius. The branes are placed at arbitrary locations which we shall denote by z_+ and z_-, where the positive and negative signs refer to the positive and negative tension branes respectively ($z_+ < z_-$). The "canonically normalized" radion modulus ϕ, whose kinetic term contribution to the dimensionally reduced action on the positive tension brane is given by

$$\frac{1}{2}\int d^4x\, \sqrt{g_+} g_+^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \tag{2.2}$$

is related to the proper distance $d = \Delta y$ between both branes in the following way (Goldberger and Wise, 1999):

$$\phi = (3M^3\ell/4\pi)^{1/2}\, e^{-d/\ell}.$$

Here, $M \sim TeV$ is the fundamental five-dimensional Planck mass. It is usually assumed that $\ell \sim M^{-1}$. Let us introduce the dimensionless radion

$$\lambda \equiv \left(\frac{4\pi}{3M^3\ell}\right)^{1/2} \phi = \frac{z_+}{z_-} = e^{-d/\ell},$$

which will also be refered to as *the hierarchy*. The effective four-dimensional Planck mass $m_{\rm pl}$ from the point of view of the negative tension brane is given by $m_{\rm pl}^2 = M^3\ell(\lambda^{-2} - 1)$. With $d \sim 37\ell$, λ is the small number responsible for the discrepancy between $m_{\rm pl}$ and M.

At the classical level, the radion is massless. However, as we shall see, bulk fields give rise to a Casimir energy which depends on the interbrane separation. This induces an effective potential $V_{\rm eff}(\phi)$ which by convention we take to be the energy density per unit physical volume on the positive tension brane, as a function of ϕ. This potential must be added to the kinetic term (2.2) in order to obtain the effective action for the radion:

$$S_{\rm eff}[\phi] = \int d^4x\, a_+^4 \left[\frac{1}{2} g_+^{\mu\nu} \partial_\mu\phi\partial_\nu\phi + V_{\rm eff}(\lambda(\phi))\right]. \tag{2.3}$$

In the following section, we calculate the contributions to $V_{\rm eff}$ from conformally invariant bulk fields.

3. MASSLESS SCALAR BULK FIELDS

The effective potential induced by scalar fields with arbitrary coupling to the curvature or bulk mass and boundary mass can be addressed. It reduces to a similar calculation to the minimal coupling massless field case, which is solved in (Garriga *et al.*, 2000), and correponds to bulk gravitons. However, for the sake of simplicity, we shall only consider below the contribution to $V_{\rm eff}(\phi)$ from conformally coupled massless bulk fields. Technically, this is much simpler than finding the contribution from bulk gravitons, and the problem of backreaction of the Casimir energy onto the background can be taken into consideration in this case. Here we are considering generalizations of the original RS proposal (Gherghetta and Pomarol, 2000; Pomarol, 1999, 2000) which allow several fields other than the graviton only (contributing as a minimally coupled scalar field).

A conformally coupled scalar χ obeys the equation of motion

$$-\Box_g\chi + \frac{D-2}{4(D-1)}R\chi = 0, \tag{3.1}$$

$$\Box^{(0)}\hat{\chi} = 0. \tag{3.2}$$

Here $\Box^{(0)}$ is the *flat space* d'Alembertian. It is customary to impose Z_2 symmetry on the bulk fields, with some parity given. If we choose even parity for $\hat{\chi}$, this results in Neumann boundary conditions

$$\partial_z \hat{\chi} = 0$$

at z_+ and z_-. The eigenvalues of the d'Alembertian subject to these conditions are given by

$$\lambda_{n,k}^2 = \left(\frac{n\pi}{L}\right)^2 + k^2, \tag{3.3}$$

where n is a positive integer, $L = z_- - z_+$ is the coordinate distance between both branes, and k is the coordinate momentum parallel to the branes.[2]

Similarly, we could consider the case of massless fermions in the RS background. The Dirac equation[3]

$$\gamma^n e_n^a \nabla_a \psi = 0$$

is conformally invariant (Birrell and Davies, 1982), and the conformally rescaled components of the fermion obey the flat space equation (3.2) with Neumann boundary conditions. Thus, the spectrum (3.3) is also valid for massless fermions.

3.1. Flat Spacetime

Let us now consider the Casimir energy density in the conformally related flat space problem. We shall first look at the effective potential per unit area on the brane, $\mathcal{A}$. For bosons, this is given by

$$V_0^b = \frac{1}{2\mathcal{A}} \mathrm{Tr}\ \ln\left(-\Box^{(0)}/\mu^2\right). \tag{3.4}$$

Here μ is an arbitrary renormalization scale. Using zeta function regularization (see, e.g., Ramond, 1989), it is straightforward to show that

$$V_0^b(L) = \frac{(-1)^{\eta-1}}{(4\pi)^\eta \eta!} \left(\frac{\pi}{L}\right)^{D-1} \zeta_R'(1-D). \tag{3.5}$$

Here $\eta = (D-1)/2$ and ζ_R is the standard Riemann's zeta function. The contribution of a massless fermion is given by the same expression but with opposite sign:

$$V_0^f(L) = -V_0^b(L). \tag{3.6}$$

[2] If we considered an odd parity field, then we would impose Dirichlet boundary conditions $\hat{\chi}(z_-) = \hat{\chi}(z_+) = 0$, and the set of eigenvalues would be the same except for the zero mode, which only the even field has.

[3] Here, e_n^a is the fünfbein, $n, m, \ldots$ are flat indices, $a, b, \ldots$ are "world" indices, and γ^n are the Dirac matrices. The covariant derivative can be expressed in terms of the spin connection ω_{anm} as $\nabla_a = \partial_a + \frac{1}{2}\omega_{\mathrm{anm}}\Sigma^{nm}$, where $\Sigma^{nm} = \frac{1}{4}[\gamma^n, \gamma^m]$ are the generators of the Lorentz transformations in spin $1/2$ representation.

The expectation value of the energy-momentum tensor is traceless in flat space for conformally invariant fields. Moreover, because of the symmetries of our background, it must have the form (Birrell and Davies, 1982)

$$\langle T_z^z \rangle_{\text{flat}} = (D-1)\rho_0(z), \qquad \langle T_j^i \rangle_{\text{flat}} = -\rho_0(z)\delta_j^i.$$

By the conservation of energy-momentum, ρ_0 must be a constant, given by

$$\rho_0^{\text{b,f}} = \frac{V_0^{\text{b,f}}}{2L} = \mp \frac{A}{2L^D},$$

where the minus and plus signs refer to bosons and fermions respectively, and we have introduced

$$A \equiv \frac{(-1)^\eta}{(4\pi)^\eta \eta!} \pi^{D-1} \zeta_{\text{R}}'(1-D) > 0.$$

This result (Antoniadis *et al.*, 1999; Delgado *et al.*, 1999), which is a simple generalization to codimension-1 branes embedded in higher dimensional spacetimes of the usual Casimir energy calculation, and it reproduces the same kind of behavior: the effective potential depends on the interbrane distance monotonously. So, depending on D and the field's spin, it induces an attractive or repulsive force, describing correspondingly the collapse or the indefinite separation of the branes, just as happened in the Appelquist and Chodos's calculation (Appelquist and Chodos, 1983a,b). In this case, then, the stabilization of the interbrane distance cannot be due to quantum fluctuations of fields propagating into the bulk.

3.2. AdS Spacetime

Now, let us consider the curved space case. Since the bulk dimension is odd, there is no conformal anomaly (Birrell and Davies, 1982) and the energy-momentum tensor is traceless in the curved case too. This tensor is related to the flat space one by (see, e.g., Birrell and Davies, 1982)

$$\left\langle T_\nu^\mu \right\rangle_g = a^{-D} \langle T_\nu^\mu \rangle_{\text{flat}}.$$

Hence, the energy density is given by

$$\rho = a^{-D} \rho_0. \tag{3.7}$$

The effective potential per unit physical volume on the positive tension brane is thus given by

$$V_{\text{eff}}(\lambda) = 2a_+^{1-D} \int a^D(z)\rho \, dz = \mp \ell^{1-D} \frac{A\lambda^{D-1}}{(1-\lambda)^{D-1}}. \tag{3.8}$$

Note that the background solution $a(z) = \ell/z$ has only been used in the very last step.

The previous expression for the effective potential takes into account the Casimir energy of the bulk, but it is not complete because in general the effective potential receives additional contributions from both branes. We can always add to $V_{\rm eff}$ terms which correspond to finite renormalization of the tension on both branes. These are proportional to λ^0 and λ^{D-1}. The coefficients in front of these two powers of λ cannot be determined from our calculation and can only be fixed by imposing suitable renormalization conditions which relate them to observables. Adding those terms and particularizing to the case of $D = 5$, we have

$$V_{\rm eff}(\lambda) = \mp \ell^{-4}\left[\frac{A\lambda^4}{(1-\lambda)^4} + \alpha + \beta\lambda^4\right], \tag{3.9}$$

where $A \approx 2.46 \times 10^{-3}$. The values of α and β can be obtained from the observed value of the "hierarchy," $\lambda_{\rm obs}$, and the observed value of the effective four-dimensional cosmological constant, which we take to be zero. Thus, we take as our renormalization conditions

$$V_{\rm eff}(\lambda_{\rm obs}) = \frac{dV_{\rm eff}}{d\lambda}(\lambda_{\rm obs}) = 0. \tag{3.10}$$

If there are other bulk fields, such as the graviton, which give additional classical or quantum mechanical contributions to the radion potential, then those should be included in $V_{\rm eff}$. From the renormalization conditions (3.10) the unknown coefficients α and β can be found, and then the mass of the radion is calculable. In Fig. 1 we plot (3.9) for a fermionic field and a chosen value of $\lambda_{\rm obs}$.

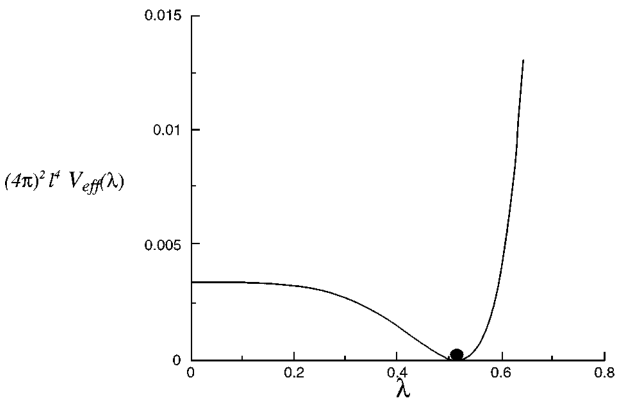

Fig. 1. Contribution to the radion-effective potential from a massless bulk fermion. This is plotted as a function of the dimensionless radion $\lambda = e^{-d/\ell}$, where d is the physical interbrane distance. The renormalization conditions (3.10) have been imposed in order to determine the coefficients α and β which appear in (3.9).

From (3.10), we have

$$\beta = -A(1 - \lambda_{\text{obs}})^{-5}, \qquad \alpha = -\beta\lambda_{\text{obs}}^5. \tag{3.11}$$

These values correspond to changes $\delta\sigma_\pm$ on the positive and negative brane tensions, which are related by the equation

$$\delta\sigma_+ = -\lambda_{\text{obs}}^5 \delta\sigma_-. \tag{3.12}$$

As we shall see below, Eq. (3.12) is just what is needed in order to have a static solution according to the five-dimensional equations of motion, once the Casimir energy is included.

We can now calculate the mass of the radion field $m_\phi^{(-)}$ from the point of view of the negative tension brane. For $\lambda_{\text{obs}} \ll 1$ we have

$$m_\phi^{2(-)} = \lambda_{\text{obs}}^{-2} m_\phi^{2(+)} = \lambda_{\text{obs}}^{-2} \frac{d^2 V_{\text{eff}}}{d\phi^2} \approx \mp\lambda_{\text{obs}} \left(\frac{5\pi^3 \zeta_R'(-4)}{6M^3 l^5} \right). \tag{3.13}$$

The contribution to the radion mass squared is negative for bosons and positive for fermions. Thus, depending on the matter content of the bulk, it is clear that the radion may be stabilized because of this effect.

Note, however, that if the "observed" interbrane separation is large, then the induced mass is small. So if we try to solve the hierarchy problem geometrically with a large internal volume, then λ_{obs} is of order TeV/m_{pl} and the mass (3.13) is much smaller than the *TeV* scale. Such a light radion would seem to be in conflict with observations. In this case we must accept the existence of another stabilization mechanism (perhaps classical or nonperturbative) contributing a large mass to the radion. Of course, another possibility is to have λ_{obs} of order one, with M and ℓ of order m_{pl}, in which case the radion mass (3.13) would be very large, but then we must look for a different solution to the hierarchy problem.

3.3. Casimir Energy Backreaction

Because of the conformal invariance, it is straightforward to take into account the backreaction of the Casimir energy on the geometry. First of all, we note that the metric (2.1) is analogous to a Friedmann–Robertson–Walker metric, where the nontrivial direction is spacelike instead of timelike. The dependence of a on the transverse direction can be found from the Friedmann equation

$$\left(\frac{a'}{a}\right)^2 = \frac{16\pi G_5}{3}\rho - \frac{\Lambda}{6}. \tag{3.14}$$

Here a prime indicates derivative with respect to the proper coordinate y [see Eq. (2.1)], and $\Lambda < 0$ is the background cosmological constant. Combined with (3.7), which relates the energy density ρ to the scale factor a, Eq. (3.14) becomes a first-order ordinary differential equation for a. We should also take into account the matching conditions at the boundaries

$$\left(\frac{a'}{a}\right)_{\pm} = \frac{\mp 8\pi G_5}{6}\sigma_{\pm}. \tag{3.15}$$

A static solution of Eqs. (3.14) and (3.15) can be found by a suitable adjustment of the brane tensions. Indeed, since the branes are flat, the value of the scale factor on the positive tension brane is conventional and we may take $a_+ = 1$. Now, the tension σ_+ can be chosen quite arbitrarily. Once this is done, Eq. (3.15) determines the derivative a'_+, and Eq. (3.14) determines the value of ρ_0. In turn, ρ_0 determines the co-moving interbrane distance L, and hence the location of the second brane. Finally, integrating (3.14) up to the second brane, the tension σ_- must be adjusted so that the matching condition (3.15) is satisfied. Thus, as with other stabilization scenarios, a single fine-tuning is needed in order to obtain a vanishing four-dimensional cosmological constant.

This is in fact the dynamics underlying our choice of renormalization conditions (3.10), which we used in order to determine α and β. Indeed, let us write $\sigma_+ = \sigma_0 + \delta\sigma_+$ and $\sigma_- = -\sigma_0 + \delta\sigma_-$, where $\sigma_0 = (3/4\pi\ell G_5)$ is the absolute value of the tension of the branes in the zeroth-order background solution. Eliminating a'/a from (3.15) and (3.14), we easily recover the relation (3.12), which had previously been obtained by extremizing the effective potential and imposing zero effective four-dimensional cosmological constant (here, $\delta\sigma_\pm$ is treated as a small parameter, so that extremization of the effective action coincides with extremization of the effective potential on the background solution). In that picture, the necessity of a single fine-tuning is seen as follows. The tension on one of the walls can be chosen quite arbitrarily. For instance, we may freely pick a value for β, which renormalizes the tension of the brane located at z_-. Once this is given, the value of the interbrane distance $\lambda_{\rm obs}$ is fixed by the first of Eqs. (3.11). Then, the value of α, which renormalizes the tension of the brane at z_+, must be fine-tuned to satisfy the second of Eqs. (5.8).

Equations (3.14) and (3.15) can of course be solved nonperturbatively. We may consider, for instance, the situation where there is no background cosmological constant ($\Lambda = 0$). In this case we easily obtain

$$a^3(z) = \frac{6\pi G A}{(z_- - z_+)^5}(C - z)^2 = \frac{3}{4}\pi^3\zeta'_{\rm R}(-4)G_5\frac{(z_0 - z)^2}{(z_- - z_+)^5}, \tag{3.16}$$

where the brane tensions are given by

$$2\pi G\sigma_\pm = \pm(C - z_\pm)^{-1}$$

and C is a constant. This is a self-consistent solution where the warp in the extra dimension is entirely due to the Casimir energy.

Of course, the conformal interbrane distance $(z_- - z_+)$ is different from the physical distance d, although they are related. For instance, imposing $a(z_+) = 1$, we can rewrite as

$$6\pi A G_5 = \left(\frac{z_- - z_+}{z_0 - z_+}\right)^2 (z_- - z_+)^3$$

and we get the relation

$$d = (z_- - z_+)\left[\frac{3}{5}\sqrt{\frac{(z_- - z_+)^3}{6\pi G_5 A}}\left(1 - \left(1 - \sqrt{\frac{6\pi G_5 A}{(z_- - z_+)^3}}\right)^{5/3}\right)\right].$$

Here we can see that when the effect of the Casimir energy is small (and so is the curvature consequently), $6\pi G_5 A/(z_- - z_+)^3 \ll 1$ indeed corresponds to the flat case, in which the conformal and the physical distances coincide.

We can also integrate Eq. (3.14) in the general case (Pujolàs, 2000) and get

$$a(y) = \left(\frac{16\pi A M^3}{-\Lambda(z_- - z_+)^5}\right)^{1/5} \sinh^{2/5}\left(\frac{5}{2}\sqrt{-\Lambda/6(y_0 - y)}\right), \tag{3.17}$$

with brane tensions given by

$$\sigma_\pm = \pm\frac{3}{4\pi}\frac{\sqrt{-\Lambda/6}}{G_5}\coth\left(\frac{5}{2}\sqrt{-\Lambda/6}(y_0 - y_\pm)\right).$$

Here we are assuming $\Lambda < 0$ and y_0 is an integration constant. Moreover this we can explicitly check how this reduces to RS solution in the limit of small Casimir energy compared to the cosmological constant, i.e., when

$$\frac{16\pi G_5}{3}\rho_0 \ll \frac{\Lambda}{6}.$$

Again fixing $a(z_+) = 1$ we find

$$y_0 = \frac{2}{5}\sqrt{\frac{-6}{\Lambda}}\,\mathrm{arcsinh}\left(\left(\frac{32\pi\rho_0}{-\Lambda M^3}\right)^{-1/2}\right) \to \infty$$

since $(32\pi\rho_0/(-\Lambda M^3)) \to 0$, so that we can write the warp factor as a series in powers of the parameter $(32\pi\rho_0/(-\Lambda M^3))^{1/5} \ll 1$:

$$a(y) \approx e^{-\sqrt{-\Lambda/6}y}\left(1 - \frac{1}{5}\left(\frac{128\pi\rho_0}{-\Lambda M^3}\right)^{2/5} e^{2\sqrt{-\Lambda/6}\,y} + \cdots\right).$$

4. CONCLUSIONS AND DISCUSSION

We have shown that in brane-world scenarios with a warped extra dimension, it is in principle possible to stabilize the radion ϕ through the Casimir force induced by bulk fields. Specifically, conformally invariant fields induce an effective potential of the form (3.9) as measured from the positive tension brane. From the point of view of the negative tension brane, this corresponds to an energy density per unit physical volume of the order

$$V_{\text{eff}}^{-} \sim m_{\text{pl}}^{4}\left[\frac{A\lambda^{4}}{(1-\lambda)^{4}} + \alpha + \beta\lambda^{4}\right],$$

where A is a calculable number (of order 10^{-3} per degree of freedom) and $\lambda \sim \phi/(M^3\ell)^{1/2}$ is the dimensionless radion. Here M is the higher dimensional Planck mass and ℓ is the AdS radius, which are both assumed to be of the same order, whereas m_{pl} is the lower dimensional Planck mass. In the absence of any fine-tuning, the potential will have an extremum at $\lambda \sim 1$, where the radion may be stabilized (at a mass of order m_{pl}). However, this stabilization scenario without fine-tuning would not explain the hierarchy between m_{pl} and *TeV*.

A hierarchy can be generated by adjusting β according to (3.11), with $\lambda_{\text{obs}} \sim (TeV/m_{\text{pl}}) \sim 10^{-16}$ (of course, one must also adjust α in order to have vanishing four-dimensional cosmological constant). But with these adjustements, the mass of the radion would be very small, of order

$$m_{\phi}^{2(-)} \sim \lambda_{\text{obs}} M^{-3}\ell^{-5} \sim \lambda_{\text{obs}}(TeV)^{2}. \tag{4.1}$$

Therefore, in order to make the model compatible with observations, an alternative mechanism must be invoked in order to stabilize the radion, giving it a mass of order *TeV*.

(Goldberger and Wise, 1999, 2000), for instance, introduced a field v with suitable classical potential terms in the bulk and on the branes. In this model, the potential terms on the branes are chosen so that the active movement within the field in the positive tension brane v_+ is different from the active movement on the negative tension brane v_-. Thus, there is a competition between the potential energy of the scalar field in the bulk and the gradient which is necessary to go from v_+ to v_-. The radion sits at the value where the sum of gradient and potential energies is minimized. This mechanism is perhaps somewhat ad hoc, but it has the virtue that a large hierarchy and an acceptable radion mass can be achieved without much fine-tuning. It is reassuring that in this case the Casimir contributions, given by (4.1), would be very small and would not spoil the model.

The graviton contribution to the radion effective potential can be computed as well. Each polarization of the gravitons contributes as minimally coupled massless bulk scalar field (Tanaka, manuscript in preparation), and since gravitons are not conformally invariant, the calculation is considerably more complex. A suitable method has been developed for this purpose (Garriga *et al*., 2000). The result is that gravitons contribute a negative term to the radion mass squared, but this term is even smaller than (4.1), by an extra power of $\lambda_{\rm obs}$. More over this method works also in AdS space for scalar fields of any kind (massive, nonminimally coupled).

In an interesting recent paper, Fabinger and Hořava (2000) have considered the Casimir force in a brane-world scenario similar to the one discussed here, where the internal space is topologically S^1/Z_2. In their case, however, the gravitational field of the branes is ignored and the extra dimension is not warped. As a result, their effective potential is monotonic and stabilization does not occur (at least in the regime where the one-loop calculation is reliable, just like in the original Kaluza–Klein compactification on a circle (Appelquist and Chodos, 1983a,b). The question of gravitational backreaction of the Casimir energy onto the background geometry is also discussed in (Fabinger and Hořava, 2000). Again, since the gravitational field of the branes is not considered, they do not find static solutions. This is in contrast with our case, where static solutions can be found by suitable adjustment of the brane tensions.

Finally, it should be pointed out that the treatment of backreaction (here and in Fabinger and Hořava, 2000), applies to conformally invariant fields but not to gravitons. Gravitons are similar to minimally coupled scalar fields, for which it is well known that the Casimir energy density diverges near the boundaries (Birrell and Davies, 1982). Therefore, a physical cutoff related to the brane effective width seems to be needed so that the energy density remains finite everywhere. Presumably, our conclusions will be unchanged, provided that this cutoff length is small compared with the interbrane separation, but further investigation of this issue would be interesting.

It seems also interesting to clarify whether the same stabilization mechanism works in other kind of warped compactified brane-world models, such as some coming form M-theory (Lukas *et al*., 1999). In this case the bulk instead of a slice of AdS (which is maximally symmetric) consists of a power-law warp factor, and consequently a less symmetric space. This complicates the calculation since, for instance, there are two four-dimensional massless moduli fields (apart from the four-dimensional gravitons) to stabilize.

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